Multidimensional Integrability via Geometry

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A CENTURY OF NOETHER'S THEOREM AND BEYOND

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Linear Lax pairs and *nonlinear* integrable systems

Linear PDEs $L\psi = 0$ are fairly well understood incl.

- behavior of solutions (asymptotics, etc.)
- explicit exact solutions (in some cases)

P.D. Lax (1960s): *a pair* of lin. part. diff. ops L & M: $[L, M] = 0 \Rightarrow$ *nonlin.* system for the coeffs of L & M that, under certain technical conditions, is called **(Lax) integrable** and has inter alia

- infinitely many explicit exact solutions
- infinitely many cons. laws & (higher) symmetries

Lin. system $L\psi = 0, M\psi = 0$ is then called a **Lax pair** for this nonlinear system; $[L, M] = 0$ is a **Lax-type representation** for the latter.

KdV equation: the prototypic integrable system

The Korteweg–de Vries equation for $u = u(x, t)$,

$$u_t + 6uu_x + u_{xxx} = 0, \qquad (1)$$

has a Lax-type representation $[L, M] = 0$ with

$$L = -\partial_x^2 - u - \lambda, \quad M = \partial_t + 4\partial_x^3 + 6u\partial_x + 3u_x.$$

$[L, M] = 0 \Rightarrow$ compatibility of Lax pair for $\psi(x, t, \lambda)$:

$$Q\psi = \lambda\psi, \qquad M\psi = 0, \qquad (2)$$

where $Q = -\partial_x^2 - u$ and λ is the *spectral parameter*

$\Rightarrow$ infinitely many conservation laws and symmetries

$\Rightarrow$ infinitely many exact solutions (incl. the famous multisolitons) via the inverse scattering transform.

Why should we care about integrable systems?

Integrable systems are both rare and universal. Like prime numbers, their rarity is due to the intricate mathematical structures underlying them, while at the same time these structures explain their universality.

Preface of the book
Discrete Systems and Integrability
by J. Hietarinta, N. Joshi, and F. W. Nijhoff

Hydrodynamic-type (a.k.a. dispersionless) systems

A partial differential system is of **hydrodynamic type** (a.k.a. **dispersionless**) if it can be written as a first-order homogeneous quasilinear system, that is,

$$A_0(\boldsymbol{u})\boldsymbol{u}_{x^0} + A_1(\boldsymbol{u})\boldsymbol{u}_{x^1} + \cdots + A_{d-1}(\boldsymbol{u})\boldsymbol{u}_{x^{d-1}} = 0; \quad (3)$$

A_i are $M \times N$ matrices, $M \geqslant N$, $\boldsymbol{u} \equiv (u^1, \ldots, u^N)^T$; $\vec{x} = (x^0, \ldots, x^{d-1})^T$, $\boldsymbol{u} = \boldsymbol{u}(\vec{x})$.

Such systems often occur in fluid mechanics $\Rightarrow$ the term *hydrodynamic-type system*

Notation: $k\mathrm{D}$ denotes k independent variables a.k.a. k dimensions, e.g. $3\mathrm{D}$ for $k = 3$ and $4\mathrm{D}$ for $k = 4$

Why *dispersionless*?

dKP system KP system

$$\left.\begin{array}{l} u_y - v_x = 0, \\ u_t - 3v_y + 6uu_x = 0 \end{array}\right\} \underset{DL}{\overset{DD}{\rightleftarrows}} \left\{\begin{array}{l} u_y - v_x = 0, \\ u_t - 3v_y + 6uu_x + \varepsilon^2 u_{xxx} = 0 \end{array}\right.$$

DD is *dispersive deformation*
DL is *dispersionless limit*
(d)KP is *(dispersionless) Kadomtsev–Petviashvili*

Warning: not all dispersionless systems admit dispersive deformations and can be written as dispersionless limits of dispersive systems

Example: a class of dispersionless quasilinear second-order equations

A quasilinear second-order PDE in d dimensions

$$\sum_{i=0}^{d-1}\sum_{j=i}^{d-1} f_{ij}(w_{x^0},\dots,w_{x^{d-1}})w_{x^i x^j}=0$$

can be written in dispersionless form with $N=d$ for $\boldsymbol{u}=(u^1,\dots,u^N)^T$ and $u^j=w_{x^{j-1}}$ as

$$\begin{cases} \sum\limits_{i=0}^{d-1}\sum\limits_{j=i}^{d-1} f_{ij}(u^1,\dots,u^d)u^i_{x^j}=0, \\ (u^i)_{x^{j-1}}=(u^j)_{x^{i-1}}, \quad i=1,\dots,d,\ j=i+1,\dots,d. \end{cases}$$

Examples of integrable dispersionless equations of the form $\sum_{i=0}^{d-1}\sum_{j=i}^{d-1} f_{ij}(w_{x^0},\dots,w_{x^{d-1}})w_{x^i x^j}=0$

- potential dKP eqn $w_{xt}+3(w_x w_{xx}-w_{yy})=0$ arising e.g. in nonlin. acoustics & fluid dynamics
- Martínez Alonso–Shabat eqn $w_{ty}=w_z^2(w_y/w_z)_x$
- 6D eqn (Sergyeyev JMAA 2017) $w_s(w_{zt}-w_{xy})+w_z(w_{ry}-w_{st})+w_y(w_{sx}-w_{rz})=0$
- such equations were studied by many authors incl. H. Baran, L. Bogdanov, B. Doubrov, E.V. Ferapontov, P. Holba, K. Khusnutdinova, I.S. Krasil'shchik, B. Kruglikov, O.I. Morozov, V.S. Novikov, M.V. Pavlov, S.P. Tsarev, P. Vojčák etc.

Lax pairs for 3D dispersionless systems (following V.E. Zakharov et al.)

Many integrable 3D dispersionless systems

$$A_0(\boldsymbol{u})\boldsymbol{u}_t + A_1(\boldsymbol{u})\boldsymbol{u}_x + A_2(\boldsymbol{u})\boldsymbol{u}_y = 0 \qquad (4)$$

have Lax-type reps $[\partial_y - \mathcal{X}_f, \partial_t - \mathcal{X}_g] = 0$ & Lax pairs

$$\chi_y = \mathcal{X}_f(\chi),\ \chi_t = \mathcal{X}_g(\chi) \text{ for } \chi = \chi(x, y, t, p). \quad (5)$$

- $\mathcal{X}_h = h_p\partial_x - h_x\partial_p$ is the *Hamiltonian vector field* in one d.o.f. with the Hamiltonian $h(p, \boldsymbol{u})$
- $f = f(p, \boldsymbol{u})$, $g = g(p, \boldsymbol{u})$ are the *Lax functions*
- *nonisospectrality*: (5) involves χ_p
- p is the *variable spectral parameter* $(\boldsymbol{u}_p \equiv 0)$

Example: the dKP system and its integrability

Dispersionless Kadomtsev–Petviashvili (dKP) system

$$u_y = v_x, \quad u_t = 3v_y - 6uu_x \tag{6}$$

- admits a Lax pair of the type (5):

$$\chi_y = \mathcal{X}_f(\chi),\ \chi_t = \mathcal{X}_g(\chi),$$

$$f = u - p^2,\ g = 4p^3 - 6up + 3v,\ \mathcal{X}_h = h_p\partial_x - h_x\partial_p:$$

$$\begin{aligned} \chi_y &= -p\chi_x - u_x\chi_p, \\ \chi_t &= (12p^2 - 6u)\chi_x - (3v_x - 6pu_x)\chi_p. \end{aligned}$$

- yields the dKP eqn $u_{tx} = 3u_{yy} - 6(uu_x)_x$ with applications in fluid dynamics & nonlin. acoustics

Integrable systems in 4D known so far

Integrable 4D systems include

- (anti-)self-dual Yang–Mills equations
- (anti-)self-dual vacuum Einstein and related eqs
  (e.g. heavenly, Przanowski and Dunajski equations,
  and equations for (A)SD conformal structures in 4D)
- a number of other isolated examples
  (L. Bogdanov, D. Calderbank, B. Doubrov, E. Ferapontov,
  B. Konopelchenko, B. Kruglikov, O. Morozov, V. Novikov,
  G. Ortenzi, P. Santini, W. Schief, M. Sheftel, I. Strachan,
  K. Takasaki, D. Yazıcı, etc.)

Big picture:

- systems in question are mostly dispersionless
  (can be written as 1st order quasilin. homogeneous systems)
- **missing** effective systematic construction like in 3D via Lax pairs with Hamiltonian vector fields

Integrable systems in 3D vs 4D: what was known to date

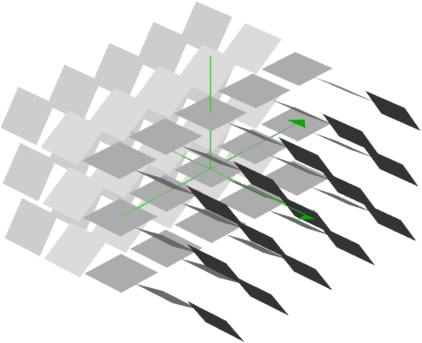
	Dispersive	**Dispersionless**
3D	systematic construction (central extension) +sporadic examples	systematic construction (Hamiltonian vec. fields) +sporadic examples
4D	exceptional examples	sporadic examples

Integrable systems in 3D vs 4D: how things really are

	Dispersive	**Dispersionless**
3D	systematic construction (central extension) +sporadic examples	systematic construction (Hamiltonian vec. fields) +sporadic examples
4D	exceptional examples	**systematic construction** (**contact vec. fields**) +sporadic examples

Standard contact structure in dimension three

Consider a 3-manifold $\mathcal{M}$ with local coordinates x, z, p and the contact one-form $\alpha = dz + pdx$ (being contact means here that $\alpha \wedge d\alpha \neq 0$ at all points of $\mathcal{M}$).



Contact vector fields

A vector field X is *contact* (w.r.t. a given contact form α) if $\exists\ f : \mathcal{M} \to \mathbb{R}$ s.t. $L_X\alpha = f\alpha$.

A contact vector field X is uniquely determined by its *contact Hamiltonian* $h_X = \alpha(X)$.

Notation: X_h is the c.v.f. with a cont. Hamiltonian h.

Contact vector fields form a Lie subalgebra in the Lie algebra of all vector fields on $\mathcal{M}$.

For $\alpha = dz + pdx$ we get

$$X_h = h_p\partial_x + (ph_z - h_x)\partial_p + (h - ph_p)\partial_z.$$

Linear contact Lax pairs

Definition *A* **linear contact Lax pair** *is a system*

$$\chi_y = X_f(\chi),\ \chi_t = X_g(\chi) \text{ for } \chi = \chi(x, y, z, t, p). \quad (7)$$

- $\boldsymbol{p}$ is the *variable spectral parameter* (recall that $\boldsymbol{u} = \boldsymbol{u}(x, y, z, t)$, so $\boldsymbol{u}_p \equiv 0$)
- $f = f(p, \boldsymbol{u})$, $g = g(p, \boldsymbol{u})$ are the *Lax functions*
- $X_h = h_p\partial_x + (ph_z - h_x)\partial_p + (h - ph_p)\partial_z$
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- $L = \partial_y - X_f$, $M = \partial_t - X_g$ are the *Lax operators*

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Remark *Lax pairs (7) provide a natural 4D generalization of well-known 3D Lax pairs (5), that is,* $\chi_y = \mathcal{X}_f(\chi)$, $\chi_t = \mathcal{X}_g(\chi)$, *where* $\mathcal{X}_h = h_p\partial_x - h_x\partial_p$, *since if* $\boldsymbol{u}_z = 0$ & $\chi_z = 0$ *then (7) boils down to (5).*

From linear contact Lax pairs to nonlinear systems

Fix f & g in (7) and consider the associated *Lax eqn*

$$[\partial_y - X_f, \partial_t - X_g] = 0, \qquad (8)$$

where $X_h = h_p\partial_x + (ph_z - h_x)\partial_p + (h - ph_p)\partial_z$.

Proposition *The Lax equation (8) holds iff so does*

$$f_t - g_y + \{f, g\} = 0, \qquad (9)$$

where $\{f, g\} = f_p g_x - g_p f_x - p\,(f_p g_z - g_p f_z) + f g_z - g f_z$.

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Equating to zero the coeffs at p^k $\forall k \in \mathbb{Z}$ in (9) $\overset{\boldsymbol{u}_p=0}{\Longrightarrow}$ a system for $\boldsymbol{u}$ with Lax pair (7) & Lax-type rep (8).

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Claim $\exists\ \infty$ *many pairs* (f, g)*: systems for* $\boldsymbol{u}$ *with Lax pairs (7) are new genuinely 4D integrable nonlin. systems transformable into Cauchy–Kowalewski form.*

From linear contact Lax pairs to differential coverings

Proposition

A hydrodynamic-type system

$$A_0(\boldsymbol{u})\boldsymbol{u}_t + A_1(\boldsymbol{u})\boldsymbol{u}_x + A_2(\boldsymbol{u})\boldsymbol{u}_y + A_3(\boldsymbol{u})\boldsymbol{u}_z = 0$$

has a Lax pair (7), that is, $\chi_y = X_f(\chi),\ \chi_t = X_g(\chi)$, *with given Lax functions* $f(p, \boldsymbol{u})$ *and* $g(p, \boldsymbol{u})$ *iff it has a differential covering of the form*

$$S_y = S_z f(S_x/S_z, \boldsymbol{u}), \quad S_t = S_z g(S_x/S_z, \boldsymbol{u}).$$

Coverings of a similar kind, namely, essentially nonlinear coverings with one-dimensional fiber, are sometimes called nonlinear Lax pairs and are closely related to integrability and also to e.g. exotic cohomology of symmetry algebras (cf. talk of O. Morozov).

Lin. contact Lax pairs: inverse scattering & more

Linear contact Lax pairs $\chi_y = X_f(\chi)$, $\chi_t = X_g(\chi)$

- belong to a broader class of nonisosp. Lax pairs

$$\left.\begin{array}{l}\chi_y = K_1(p,\boldsymbol{u})\chi_x + K_2(p,\boldsymbol{u})\chi_z + K_3(p,\boldsymbol{u})\chi_p, \\ \chi_t = L_1(p,\boldsymbol{u})\chi_x + L_2(p,\boldsymbol{u})\chi_z + L_3(p,\boldsymbol{u})\chi_p,\end{array}\right. \quad (*)$$

- hence are amenable to the inverse scattering transform (cf. e.g. Manakov & Santini 2014 etc.).

Lax pairs $(*)$ are related inter alia to geometry of characteristic varieties of associated integrable systems, cf. in particular the preprint

D. Calderbank & B. Kruglikov, arXiv:1612.02753

Two infinite families of nice pairs of Lax functions

Theorem *Lax pairs* $\chi_y = X_f(\chi)$, $\chi_t = X_g(\chi)$, *where* $X_h = h_p\partial_x + (ph_z - h_x)\partial_p + (h - ph_p)\partial_z$, *yield new integrable* $4D$ *systems that can be brought into Cauchy–Kowalewski form for the following pairs of* f *&* g*:*

1. $f = p^{n+1} + \sum_{i=0}^{n} u_i p^i$, $g = p^{m+1} + \frac{m}{n} u_n p^m + \sum_{j=0}^{m-1} v_j p^j$

with $\boldsymbol{u} = (u_0, \ldots, u_n, v_0, \ldots, v_{m-1})^T$;

2. $f = \sum_{i=1}^{m} \frac{a_i}{(p - u_i)}$, $\quad g = \sum_{j=1}^{n} \frac{b_j}{(p - v_j)}$

with $\boldsymbol{u} = (a_1, \ldots, a_m, u_1, \ldots, u_m, b_1, \ldots, b_n, v_1, \ldots, v_n)^T$.

Here $m, n = 1, 2, 3, \ldots$ *are arbitrary natural numbers.*

4D integrable generalization for dKP: the Lax pair

Let $f = p^2 + wp + u$, $g = p^3 + 2wp^2 + rp + v$,

i.e. $m = 2$, $n = 1$, $u_0 \equiv u, u_1 \equiv w, v_0 \equiv v, v_1 \equiv r$,

in class 1 of the above thm.

The Lax pair $\chi_y = X_f(\chi)$, $\chi_t = X_g(\chi)$ then reads

$$\begin{aligned}
\chi_y &= (2p + w)\chi_x + (-p^2 + u)\chi_z \\
&\quad + (w_z p^2 + (u_z - w_x)p - u_x)\chi_p, \\
\chi_t &= (r + 4wp + 3p^2)\chi_x + (v - 2wp^2 - 2p^3)\chi_z \\
&\quad + (2w_z p^3 + (r_z - 2w_x)p^2 + (v_z - r_x)p - v_x)\chi_p.
\end{aligned}$$

$$\mathit{Recap}: X_h = h_p\partial_x + (ph_z - h_x)\partial_p + (h - ph_p)\partial_z$$

4D integrable generalization for dKP: the system

For $f = p^2 + wp + u$ and $g = p^3 + 2wp^2 + rp + v$ Eq. (9), i.e., $f_t - g_y + \{f, g\} = 0$, upon bearing in mind that $\boldsymbol{u_p} = 0$ by assumption and equating to zero the coefficients at all powers of p, yields a system

$$
\begin{aligned}
&u_t - vu_z - ru_x + uv_z + wv_x - v_y = 0,\\
&2u_z + w_x + 2ww_z - r_z = 0,\\
&2r_x - 3u_x - 2w_y + 2wu_z - v_z - 2ww_x + 2uw_z = 0,\\
&w_t - r_y + 2v_x - 4wu_x + wr_x - rw_x - vw_z + ur_z = 0.
\end{aligned}
\tag{10}
$$

Proposition *System (10) is, up to a suitable rescaling, an* integrable *generalization to the case of* four *independent variables for the dKP system (6), i.e.,*

$$u_y = v_x, \; u_t = 3v_y - 6uu_x.$$

4D integrable generalization for dKP: reduction

If $u_z = v_z = 0$ & $w = 0$, $r = 3u/2$ then (10) yields

$$4u_t - 4v_y - 6uu_x = 0,\ 4v_x - 3u_y = 0, \qquad (*)$$

which is, up to a rescaling, nothing but the dKP (6):

$$u_y - v_x = 0,\ u_t - 3v_y + 6uu_x = 0.$$

Eliminating v from $(*) \Rightarrow 4u_{tx} - 3u_{yy} - 6(uu_x)_x = 0$; after $t \to 4t$ and $u \to -u$ we get the dKP equation

$$(u_t + 6uu_x)_x - 3u_{yy} = 0:$$

- applied e.g. in fluid dynamics & nonlin. acoustics
- modulo potentialization & rescaling also known as
  - the *Lin–Reissner–Tsien* equation, or
  - the *Khokhlov–Zabolotskaya* equation

Lax functions polynomial in p

Let m and n be arbitrary natural numbers,

$N = m + n + 1$, $\boldsymbol{u} = (u_0, \ldots, u_n, v_0, \ldots, v_{m-1})^T$,

$$f = p^{n+1} + \sum_{i=0}^{n} u_i p^i, \quad g = p^{m+1} + \frac{m}{n} u_n p^m + \sum_{j=0}^{m-1} v_j p^j.$$

Equating to zero the coefficients at all powers of p in Eq. (9), i.e., $f_t - g_y + \{f, g\} = 0$, while remembering that $\boldsymbol{u}_p = 0$ yields a dispersionless system shown at the next slide.

Recap: $\{f, g\} = f_p g_x - g_p f_x - p\left(f_p g_z - g_p f_z\right) + f g_z - g f_z$

Lax functions polynomial in p: the system

$$
\begin{aligned}
&(u_k)_t - (v_k)_y + m\,(u_{k-m-1})_z - n\,(v_{k-n-1})_z \\
&+(n+1)\,(v_{k-n})_x - (m+1)\,(u_{k-m})_x \\
&+\sum_{i=0}^{n}\Big\{(k-i-1)v_{k-i}\,(u_i)_z - (i-1)u_i\,(v_{k-i})_z \\
&-(k+1-i)v_{k+1-i}\,(u_i)_x + iu_i\,(v_{k+1-i})_x\Big\} = 0.
\end{aligned}
$$

Here $k = 0, \ldots, n+m$, $u_i \stackrel{\mathrm{def}}{=} 0$ for $i > n$ and $i < 0$, $v_j \stackrel{\mathrm{def}}{=} 0$ for $j > m$ and $j < 0$; $v_m \stackrel{\mathrm{def}}{=} (m/n)u_n$.

This system is (crypto-)evolutionary: it can be solved for the z-derivatives $(u_i)_z$ and $(v_j)_z$ for all i and j.

Lax functions rational in p

$$\forall m, n \in \mathbb{N} \text{ let } f = \sum_{i=1}^{m} \frac{a_i}{(p - u_i)}, \quad g = \sum_{j=1}^{n} \frac{b_j}{(p - v_j)},$$

$$\boldsymbol{u} = (a_1, \dots, a_m, u_1, \dots, u_m, b_1, \dots, b_n, v_1, \dots, v_n)^T.$$

Eq. (9), i.e., $f_t - g_y + \{f, g\} = 0$, yields a dispersionless system for $\boldsymbol{u}$ shown at the next slide which

- can be brought into Cauchy–Kowalewski form e.g. by passing from t to $T = y + t$ with all other variables intact
- if $\boldsymbol{u}_z = 0$ reduces to a known integrable 3D system found by V.E. Zakharov in 1994

Lax functions rational in p: the system

$$(u_i)_t + \sum_{j=1}^{n} \left\{ \left(\frac{b_j}{v_j - u_i} \right)_x - \left(\frac{b_j u_i}{v_j - u_i} \right)_z - \frac{2b_j (u_i)_z}{v_j - u_i} \right\} = 0, \quad i = 1, \ldots, m,$$

$$(v_j)_y + \sum_{i=1}^{m} \left\{ -\left(\frac{a_i}{v_j - u_i} \right)_x + \left(\frac{a_i v_j}{v_j - u_i} \right)_z + \frac{2a_i (v_j)_z}{v_j - u_i} \right\} = 0, \quad j = 1, \ldots, n,$$

$$\begin{aligned}(a_i)_t + \sum_{j=1}^{n} \Bigg\{ &\left(\frac{a_i b_j}{(v_j - u_i)^2} \right)_x + \left(\frac{a_i b_j (v_j - 2u_i)}{(v_j - u_i)^2} \right)_z \\ &+ \frac{3a_i (b_j)_z}{v_j - u_i} + \frac{3a_i b_j (v_j)_z}{(v_j - u_i)^2} \Bigg\} = 0, \quad i = 1, \ldots, m,\end{aligned}$$

$$\begin{aligned}(b_j)_y + \sum_{i=1}^{m} \Bigg\{ &\left(\frac{a_i b_j}{(v_j - u_i)^2} \right)_x + \left(\frac{a_i b_j (v_j - 2u_i)}{(v_j - u_i)^2} \right)_z \\ &+ \frac{3a_i (b_j)_z}{v_j - u_i} + \frac{3a_i b_j (v_j)_z}{(v_j - u_i)^2} \Bigg\} = 0, \quad j = 1, \ldots, n.\end{aligned}$$

Summary of main results

- We found a large new class of integrable systems in four independent variables (4D) with Lax pairs of a novel kind related to contact geometry

  $\Rightarrow$ there is significantly more integrable 4D systems than it appeared before

- This new class contains two new infinite families of 4D integrable systems:
  - associated with polynomial Lax functions of a special form including e.g. a new 4D integrable generalization for the well-known dispersionless Kadomtsev–Petviashvili equation
  - associated with rational Lax functions of a special form

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Ref: LMP 108 (2018), 359-376 (arXiv:1401.2122)

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Děkuji za pozornost